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FLOW OSCILLATIONS IN FIXED-PRESSURE-DROP FLOW-BOILING SYSTEMS WITH RANDOM EXCITATION

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Abstract—Flow oscillations in fixed-pressure-drop flow-boiling systems are very relevant to the design of boiling water nuclear reactors. Typically, the reactor geometry is such that the flow through an individual heated channel has little effect on the total pressure drop, and thus the flow is essentially a fixed-pressure drop flow.

The predictions of the flow under fixed-pressure-drop conditions leave much to be desired. Specifically, the theoretical and experimental oscillations differ in many significant respects: while the dominant frequency is well predicted, the amplitude is not, yet it is the latter which is of greatest importance in the operation of the reactor.

In the present paper it is shown that the introduction of random excitation in the wall heat-flux results in flow oscillations which agree significantly better with experimental data than do previous predictions.

NOMENCLATURE

A, cross-section area [m^2];
C_0, distribution function defined by equation (4) [dimensionless];
C_3, distribution function defined by equation (12) [dimensionless];
C_4, distribution function defined by equation (13) [dimensionless];
C_p, liquid specific heat;
D_h, hydraulic diameter [m];
f, frequency [H];
G, mass velocity [kg/m^2 s];
g, acceleration of gravity [m/s^2];
h_0, interphase heat-transfer coefficient [W/m^2 °C];
i^*, liquid phase enthalpy [dimensionless];
i_0^*, liquid phase enthalpy at start of void [dimensionless];
i_{fg}, heat of vaporization [W s/kg];
M, momentum flux [kg/m s^2];
Δp, pressure drop [Pa];
P_w, wetted perimeter [m];
$\dot{q}''_w$, wall heat flux [W/m^2];
$\dot{q}''_m$, time average wall heat flux [W/m^2];
t, time [s];
V_α, void propagation velocity [m/s];
V_i, enthalpy propagation velocity [m/s];
v, velocity [m/s];
v_i, inlet velocity [m/s].

Greek symbols

α, void [dimensionless];
Γ_v, vapour mass generation rate [kg/m^3 s];
Γ_m, equilibrium vapour mass generation rate [kg/m^3 s];
ε, statistical parameter [dimensionless];
θ, angular orientation of channel [dimensionless];
ρ_l, liquid phase density [kg/m^3];
ρ_v, vapour phase density [kg/m^3];
$\bar{\rho}$, average density [kg/m^3];
$\Delta\rho$, $\rho_l - \rho_v$ [kg/m^3];
σ^2, variance [dimensionless];
τ_w, wall shear stress [N/m^2];
Ω_α, void generation function [s^{-1}];
Ω_i, enthalpy generation function [s^{-1}].

INTRODUCTION

THE BEHAVIOUR of fixed-pressure-drop flow-boiling channels is of direct relevance to the design of boiling-water nuclear power reactors. Typically, the reactor geometry is such that the flow through any individual heated channel has little effect on the header-to-header pressure drop, and thus the flow is essentially a fixed-pressure-drop flow. Because of the nature of the coupling between the flow rate, acceleration, heat transfer, void distribution, and pressure drop, the flow under some conditions is not steady even when the boundary pressure conditions are steady. Rather the flow is oscillatory in character. As present installations are upgraded, and as newer designs are developed, operating conditions are encroaching on the region of significant flow oscillations, with all the concomitant implications for reactor control etc.

As a consequence of its practical importance in power reactor design, this matter of flow oscillations in fixed-pressure-drop flow-boiling systems has been the subject of much theoretical and experimental investigation (references [1–20] inclusive). At present the agreement between prediction and experiment is incomplete. The theoretical predictions have been typically presented in the form of a threshold heat-flux, below which the flow is steady and above which flow oscillations of large magnitude occur. Indeed, most theoretical models indicate that, above the threshold value of heat-flux, the flow-oscillations will grow with-

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out bound. To the authors' knowledge the previously predicted flow oscillations, whatever their amplitude, are of a regular periodic character; no irregular or random excursions are predicted.

As is discussed in greater detail below, the experimental data differ from the predictions in several important respects. In particular, the onset of flow oscillations is frequently not an abrupt phenomenon nor are the observed oscillations of a regular periodic nature. Rather the amplitude of the oscillations increases gradually with heat-flux, and at lower values of the heat-flux oscillations appear essentially random, becoming more regular as the heat-flux is increased. These characteristics suggest, at least superficially, the response of an inertial system with variable damping to random excitation. Thus the authors were led to investigate the response of a fixed-pressure-drop flow-boiling system to random excitation; specifically, we have examined the response of such a system to random fluctuations in the rate of vapour generation. The results of this examination which are the subject matter of the present contribution.

EXPERIMENTAL FLOW OSCILLATIONS

Typical traces of fixed-pressure-drop flow oscillations are shown in Fig. 1. The data is that of Canadian Westinghouse Ltd., as reported by Collins and Gacesa [13], and was obtained in test apparatus which closely simulated the flow in an actual power reactor pressure tube with 19-rod fuel bundles. Traces are presented for various heat-flux levels. The ordinates are the pressure drop across an inlet venturi, and thus are closely related to flow rate. The abscissa is time.

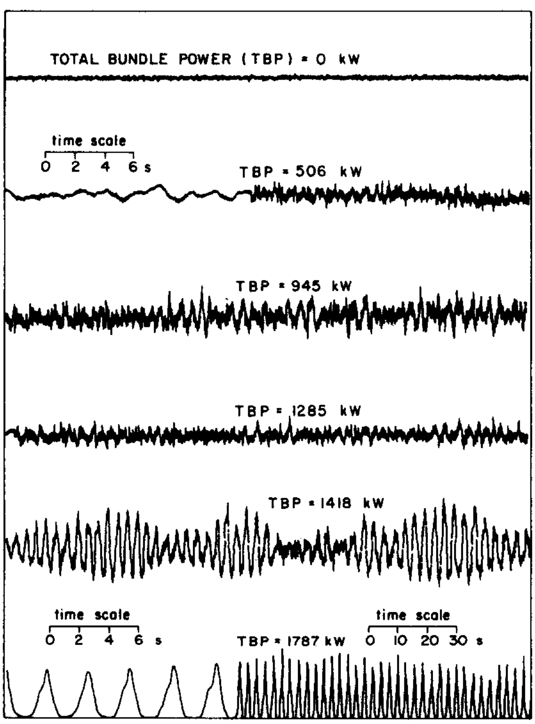

FIG. 1. Experimental flow–time variations: data of Collins and Gacesa [13]. (Ordinate scale consistent from trace to trace.)

These data display three major characteristics. First, the magnitude of the oscillations increases rather gradually with increasing heat-flux; no distinct identifiable threshold heat-flux is discernible.† Secondly, the observed fluctuations at low heat-flux levels contain a significant random component; as the heat-flux is increased a dominant frequency component emerges until, at the highest reported values of the heat-flux‡ the oscillation is of a regular periodic nature. Finally, the magnitude of the oscillations are bounded at all levels of heat-flux. This particular characteristic is displayed by all data of which the authors are aware. A fourth, and probably less significant, characteristic is that the oscillation amplitude (Fig. 1, second trace from bottom) appears to be bounded by an envelope the amplitude of which varies with a wavelength much greater than that of the dominant flow oscillation.

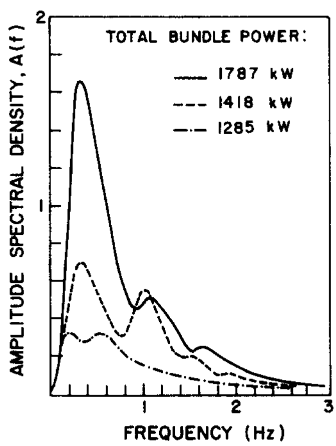

FIG. 2. Amplitude spectral density as reported by Hancox [16]. Data of Collins and Gacesa as given in Fig. 1.

The first two characteristics may perhaps be seen more clearly in Fig. 2. In this figure the amplitude spectral density of flow oscillations is plotted vs frequency. The data are those reported by Hancox [16], and were obtained by analogue processing of the data shown in Fig. 1. It is apparent from these spectra that the flow oscillations cannot in general be characterized

†This observation reflects ill on the practice of many authors of reporting only "threshold heat-fluxes" or values of the heat flux at the "threshold of flow oscillations". Without additional details, the significance of such reported values of the heat-flux may not be apparent to readers unfamiliar with complete experimental results. Even worse, such reported data may be misinterpreted as indicating a sharp, well-defined, threshold when in fact no such threshold exists. Indeed, on occasion, the amplitude is not even a monotonic function of heat-flux, as may be seen from the third and fourth traces. The authors do not, of course, wish to imply that a distinct threshold never exists, but only that the reporting of such a threshold value without further information can be inadequate.

‡In the tests with which the authors are familiar the onset of wall-temperature excursions or "dryout" determined the maximum value of the heat-flux. Under some conditions this can occasionally occur at heat-flux levels at which a significant random component is still present.

by a single threshold heat flux, however defined, nor by a single characteristic frequency. The comments above concerning the random nature of the oscillations at the lower values of heat-flux and the emergence of a dominant frequency oscillation at higher values of the heat-flux are evidenced by the relative flat spectral distributions at lower values of the heat-flux and the more peaked distribution at the highest heat-flux.

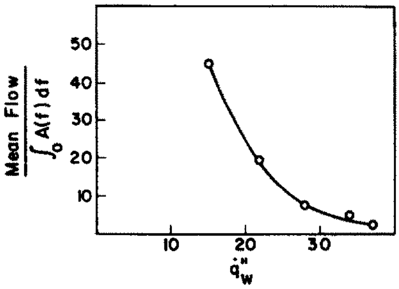

FIG. 3. Variation of oscillation amplitude with power. As reported by Hancox [16].

The relatively gradual increase of amplitude with heat-flux is further emphasized in Fig. 3 [16] in which the dimensionless inverse integral oscillation amplitude [$\equiv$ mean flow/$\int A(f)\,df$] is plotted against the heat flux. Were a distinct threshold to exist, the line through the data points should be vertical or at least quite steep. Instead, for higher values of heat-flux, the slope is quite shallow.

THE THEORETICAL MODEL

For the basic mathematical model of the flow we have chosen that of Hancox and Nicoll [17, 18]. Our reasons, apart from the obvious one of familiarity to one of the present authors, is that this model incorporates the effects of non-uniform void and velocity profiles and the effects of departures from thermal equilibrium, both of which have significant effects on the flow dynamics. Equally important, this model has been tested against a wide range of experimental data, both transient and steady-state, and has been found to yield accurate predictions [17, 18]. Finally, it has the unique, to the authors' knowledge, feature of predicting finite amplitude response to perturbations. This may be seen in Fig. 4. In view of the observations made above concerning the nature of experimental flow oscillations this feature is regarded as quite significant and indeed essential if meaningful results are to be obtained. In this connection it is appropriate to anticipate the material below and to note that the computer processing of the flow–time curve is based on the assumption that this curve is stochastic; if it is not, the statistical results are meaningless. If the theoretical model yields flow–time curves which grow without bound with increasing time, then these curves are not stochastic and the resultant spectra would indeed be physically meaningless.

Since the details of the Hancox–Nicoll model are presented in detail elsewhere, we will here confine ourselves to a brief description of the essentials only.

The continuity equations for the two phases can be combined, as shown by Zuber and Staub [21] to yield:

$$\frac{\partial\langle\alpha\rangle}{\partial t}+V_\alpha\frac{\partial\langle\alpha\rangle}{\partial z}=\Omega_\alpha \tag{1}$$

where α is the local void, Ω_α the void generation function, V_α the void propagation velocity, at t and z respectively the time and space coordinates. The symbol in angle brackets denotes the cross-section averaging operation $\int_A(\ )\,dA/A$. The expressions for V_α and Ω_α are:

$$V_\alpha=\left[C_0+\frac{\partial C_0}{\partial\langle\alpha\rangle}\langle\alpha\rangle\right]\langle v\rangle \tag{2}$$

$$\Omega_\alpha=\left[1-C_0\langle\alpha\rangle\frac{\Delta\rho}{\rho_l}\right]\frac{\langle\Gamma_v\rangle}{\rho_v} \tag{3}$$

where Γ_v is the volumetric rate of vapour of mass generation and C_0 is the distribution function which accounts for non-uniformity of velocity and void across the flow cross-section. C_0 is defined by:

$$C_0\equiv\frac{\langle\alpha v\rangle}{\langle\alpha\rangle\langle v\rangle}. \tag{4}$$

v is the local fluid velocity. The expression for $\langle v\rangle$ is:

$$\langle v\rangle=v_i(t)+\int_0^z\left[\frac{\Delta\rho}{\rho_l\rho_v}\langle\Gamma_v\rangle\right]_t dz. \tag{5}$$

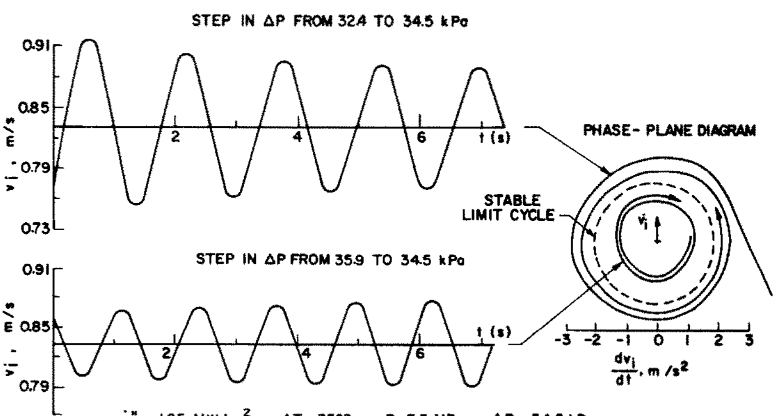

FIG. 4. Flow response and phase-plane diagrams for a step change in pressure drop. From Hancox and Nicoll [18].

If the vapour phase is assumed to be saturated, and the kinetic energy terms neglected, the energy equations may also be written as a propagation equation [16], viz:

$$\frac{\partial i^*}{\partial t} + V_i \frac{\partial i^*}{\partial z} = \Omega_i \tag{6}$$

where i^* is the dimensionless liquid phase subcooling, V_i the enthalpy propagation velocity, and Ω_i the enthalpy generation function. Expressions for the latter two quantities [17, 18] are:

$$V_i = \frac{[1 - C_0\langle\alpha\rangle]\langle v\rangle}{1 - \langle\alpha\rangle} \tag{7}$$

$$\Omega_i = -\frac{\Gamma_m - (1+i^*)\langle\Gamma_v\rangle}{\rho_l(1 - \langle\alpha\rangle)}. \tag{8}$$

Finally, the momentum equation for the mixture may be written in integral form as

$$\Delta P = -\int_0^z \left[\frac{\partial G}{\partial t} + \frac{\partial M}{\partial z} + \frac{\tau_\omega P_\omega}{A} + g\bar{\rho}\cos\theta\right] dz \tag{9}$$

where the mass velocity and momentum flux, G and M respectively are given by:

$$G = \left[1 - C_0\langle\alpha\rangle\frac{\Delta\rho}{\rho_l}\right]\langle v\rangle \tag{10}$$

$$M = \left[C_4 - C_3\langle\alpha\rangle\frac{\Delta\rho}{\rho_l}\right]\rho_l\langle v\rangle^2. \tag{11}$$

The distribution functions C_3 and C_4 account for the effects of non-uniform void and velocity profiles. Their definitions are:

$$C_3 \equiv \langle\alpha v^2\rangle/\langle\alpha\rangle\langle v\rangle^2 \tag{12}$$

$$C_4 \equiv \langle v^2\rangle/\langle v\rangle^2. \tag{13}$$

Given suitable expressions for C_0, C_3, and C_4; and for the wall shear stress τ_ω and the vapour generation rate $\langle\Gamma_v\rangle$, equations (1), (6) and (9) can be numerically integrated. For the present purpose we note the central role played by the vapour generation rate $\langle\Gamma_v\rangle$. The expression for this quantity given by Hancox and Nicoll [17, 18] was:†

$$\langle\Gamma_v\rangle = \begin{cases}\Gamma_m[1-(1-\langle\alpha\rangle)^{\frac{1}{2}} i^*/i_0^*] & i^* < i_0^* \\ 0 & i > i_0^*\end{cases} \tag{14}$$

where Γ_m is the thermal equilibrium rate of vapour generation. The expression for i_0^* is:

$$i_0^* = \dot{q}''_w C_p / h_0 i_{fg} \tag{15}$$

where h_0 is the interphase heat-transfer coefficient. The expression for Γ_m is:

$$\Gamma_m = \frac{4''\dot{q}_w}{D_n i_{fg}}. \tag{16}$$

The central role played by the wall heat flux, $\dot{q}''_w$ is apparent. Further, the expressions given by the original references implicitly assume that the vapour generation rate is constant if the wall heat flux is constant. The present authors have modified the vapour generation function by assuming the wall heat flux to be given by

$$\dot{q}''_w = \dot{q}''_m(1+\varepsilon) \tag{17}$$

where $\dot{q}''_m$ is the time mean heat flux and ε is a random fluctuation in time with a specified variance σ^2. This is the essential modification in the present work. In conducting the calculations, a new value of ε was chosen from a random number generator† every other step. This choice of maintaining ε constant for two time steps was indicated by convergence considerations in the iterative solution technique required by the integral momentum equation. In all cases the frequency of change of the value of ε was much greater than the significant frequencies in the flow oscillations (see Figs. 6–8).

Since this assumption of random variations in the heat flux is the key assumption in the present work, it is appropriate to discuss its significance in somewhat greater detail. Two aspects of the assumption warrant scrutiny. They are: is the manner in which the fluctuations are introduced, that is, through the wall heat flux, reasonable; and are random variations appropriate or should perhaps a more ordered variation be chosen?

With regard to the reasonableness of introducing fluctuations in the heat flux term, the authors feel that in the light of our present limited knowledge of the vapour generation in flow-boiling, it is indeed reasonable. Anyone who has visually observed the boiling process knows that vapour is not, in general, generated at a uniform rate, even if all reasonable precautions have been taken to ensure a constant heat inflow to the solid–fluid interface. It may be preferable to some to introduce the fluctuation directly into the vapour generation rate $\langle\Gamma_v\rangle$. Since at present there exists no sound basis for determining *a priori* the dependence of the variance, σ^2, on the other parameters, there is little difference in the end result in the two alternative procedures. In the present case it was a matter of convenience to choose to introduce the fluctuation in the heat flux term.

With regard to the choice of random variation as opposed to a more ordered alternative, very little can be said. As our knowledge of the flow-boiling process increases, it may indeed become preferable to choose some alternative. At present, however, the only guidance the authors had was a preliminary investigation [19] of a simplified flow-boiling model subjected to random fluctuations in the vapour generation rate; this model did yield flow oscillations with many of the characteristics of observed flow oscillations.

Finally, we should note that in the real situation other sources of excitation may indeed be significant. One such source, for example, might be pressure fluctuations transmitted from pumps, etc. in the system. Such alternative sources have not been investigated in the present contributions; it may, however, be per-

†A simpler expression is given by Seiveright *et al.* [22].

†This algorithm was available from the Users library of the University of Waterloo's computing centre.

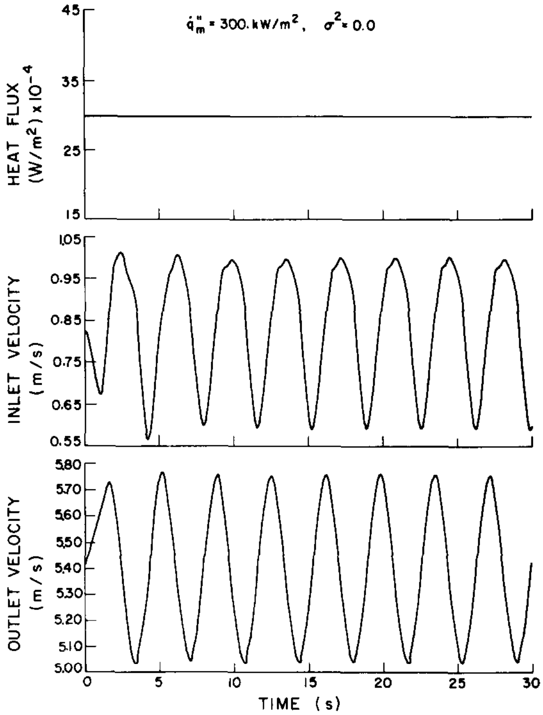

FIG. 5. Predicted flow oscillations.

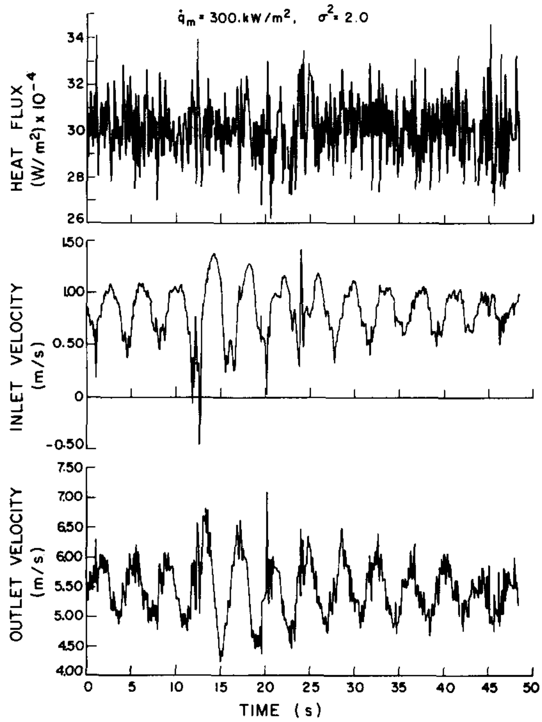

FIG. 7. Predicted flow oscillations.

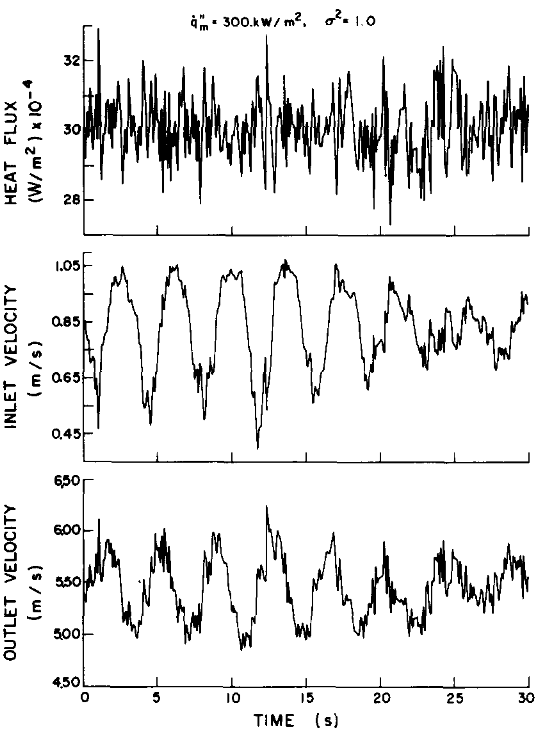

FIG. 6. Predicted flow oscillations.

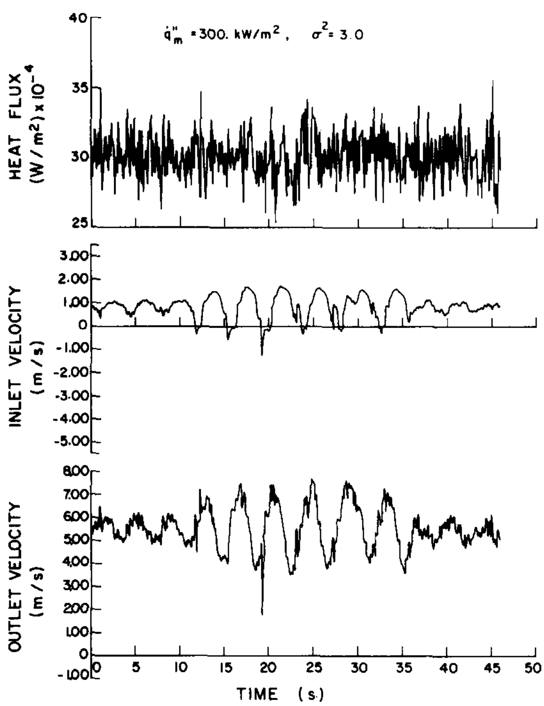

FIG. 8. Predicted flow oscillations.

missible to speculate that the resultant flow oscillations would be similar to those obtained with the present model, since imposed pressure fluctuations may be expected to influence the rate of vapour generation.

RESULTS AND DISCUSSION

Some results obtained as discussed above are presented in Figs. 5–8 inclusive. In these figures the variation with time of the wall heat-flux, inlet and outlet velocities, are shown. The system parameters chosen correspond to those reported by D'Arcy† [14]. A wide range of heat-flux values and σ^2 values are covered in the calculations, much more than can be presented

†D'Arcy defined the threshold heat flux as the value when the dominant frequency component just became apparent on the flow traces.

here. The interested reader is referred to the M.A.Sc. thesis of Brimley [23] in which the complete set of results is presented, as well as detailed descriptions of all numerical procedures. For the present paper we will examine only a limited selection of Brimley's results. In order to put these results into perspective we should note that D'Arcy [14] reported the value of heat-flux at the oscillation threshold of $475\,kW/m^2$ and an oscillation frequency of 0.34 Hz.

In performing these calculations the authors required a set of initial conditions. These were obtained from a steady state calculation for the same pressure drop and for constant heat-flux. This choice of initial conditions, while adequate for the task at hand, is arbitrary. Further, it can introduce a spurious transient which may result in a spurious dominant frequency component. This point is illustrated graphically in Figs. 4 and 5. In Fig. 4 we observe that the behaviour of the system appears to be a limit cycle, independent of the initial condition. At the start of the oscillation, however, there exists a transient which is dependent on the exact initial conditions.† This initial transient may be seen also in Fig. 5. The only method of eliminating this effect would be to allow the computer program to continue running for a sufficient time period for the transient effect to contribute negligibly to the overall result. Unfortunately, this is uneconomical, and while we have attempted to minimize any effects of this spurious transient, the reader should be aware of the problem, and should be cautious in interpreting the results in the vicinity of the dominant frequency.

Oscillatory behaviour for a constant wall heat flux and vapour generation ($\sigma^2 = 0$) is shown in Fig. 5. The oscillation is quite regular and is quite different in character from experimental data. The corresponding results for fluctuating vapour generation are shown in Figs. 6, 7 and 8. Here we may observe the characteristics noted in the previous discussion of experimental data.

The character of the predicted flow oscillations is shown more clearly in Figs. 9 and 10. In these figures the amplitude spectral density is plotted against frequency. These figures were obtained using a fast Fourier transform method developed by Cooley and Tukey [24]. For details the reader is again asked to refer to Brimley [23].

In Fig. 9 the effect of increasing the variance is shown. As expected, the effect is an increase of amplitude with increasing variance. Somewhat less obvious is that the increase in amplitude is much greater, on a percentage basis, than the increase in variance. Figure 9 shows a relatively minor change in amplitude at the dominant frequency. Without further computations it is not possible to state with certainty just what fraction of this behaviour is due to the transient.

In Fig. 10 the effects of increasing heat flux (with a variance of fixed proportion of the mean) are shown. Initially the increase is very large, but the difference

†This suggests that determinations of system stability based on the growth or shrinkage of amplitude in the first few cycles should be viewed with caution. Unfortunately, this practice is common.

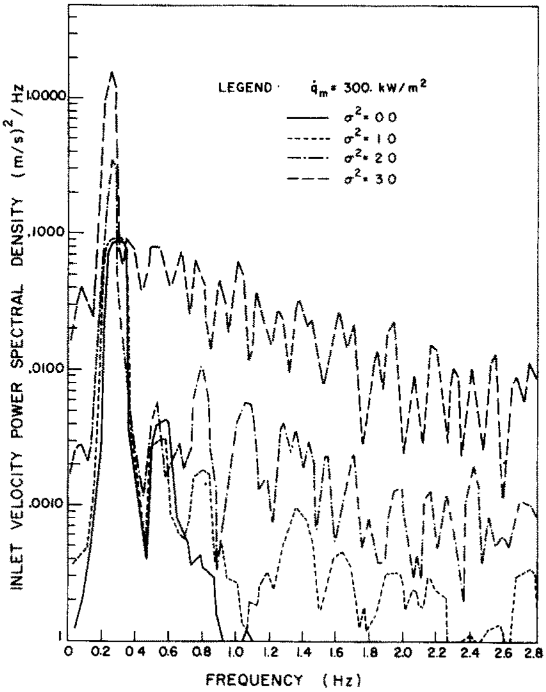

FIG. 9. Predicted flow oscillation spectra.

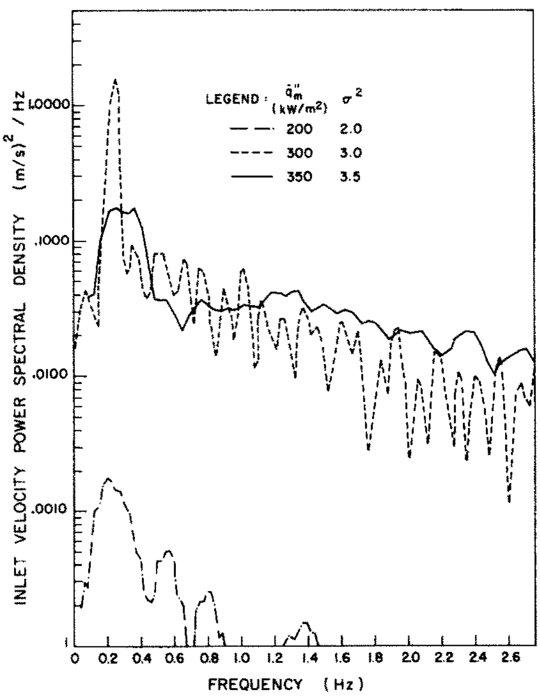

FIG. 10. Predicted flow oscillation spectra.

between the graphs for $\dot{q}''_m$ of 300 and $350\,kW/m^2$ is much less so. This again is consistent with the character of experimental oscillations discussed above. Figure 10 is also interesting in that it displays clearly the effect of spurious transients. Specifically, the difference between the graphs for $\dot{q}''_m$ of 300 and $350\,kW/m^2$ around a frequency of approximately 0.3 Hz, shows this effect. While "smoothing" in the determination of the spectra would have disguised this, the authors felt it useful not to do so. In this connection it is useful to remind the reader that in a numerical determination of spectra from a finite set of data, the degree of smoothing is arbitrary and represents a compromise between the conflicting requirements of resolution and precision.

CONCLUSIONS

In the previous sections we have discussed in some detail the characteristics of experimental flow oscillations and the difference between the predictions of various theoretical models and these data. The present work has shown that these differences can be greatly reduced if excitation is introduced into the theoretical model through random fluctuations in the wall heat-flux. Specifically, the relatively gradual increase of amplitude with increasing heat flux, and the spectral distributions are similar. Since the present model involves the variance of the heat flux fluctuations as a parameter, it is not at present possible to go further and to compare prediction and experiment; before meaningful comparison can be made much more data must be obtained. Specifically, data is required which would allow the variance to be related to other system flow and heat-transfer characteristics. The present work does suggest that this data should include, at the very least, flow oscillation spectra at various power levels.

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MOUVEMENTS D'OSCILLATION DU FLUIDE DANS LES SYSTEMES D'ECOULEMENT EN EBULLITION AVEC CHUTE DE PRESSION CONSTANTE ET EXCITATIONS ALEATOIRES

Résumé—Les oscillations du fluide dans les systèmes d'écoulement en ébullition avec chute de pression constante sont importantes dans la conception de réacteurs nucléaires à eau bouillante. La géométrie d'un réacteur dans les cas types est telle que l'écoulement dans chaque canal chauffé a peu d'effet sur la différence totale de pression, et il s'en suit que l'écoulement est essentiellement à chute de pression constante.

Les prévisions de l'écoulement dans des conditions de chute de pression constante laissent beaucoup à désirer. En particulier, les oscillations théoriques et expérimentales différent sous bien des aspects importants; la fréquence fondamentale est prévue correctement, mais l'amplitude ne l'est pas; cependant cette dernière est la plus importante dans le fonctionnement du réacteur.

Dans le présent article on montre que l'introduction d'excitations aléatoires dans le flux thermique pariétal engendre des oscillations de l'écoulement qui sont en bien meilleur accord avec les données expérimentales que ne l'étaient les prévisions précédentes.

STRÖMUNGSOSZILLATIONEN IN SIEDESYSTEMEN MIT KONSTANTEM DRUCKABFALL UND ZUFALLSVERTEILTEN STÖRGRÖSSEN

Zusammenfassung—Strömungsoszillationen beim Strömungssieden in Systemen mit konstantem Druckabfall sind von großer Bedeutung beim Entwurf von Siedewasserreaktoren. Die Reaktorgeometrie ist gewöhnlich derart, daß die Strömung durch einen individuell beheizten Kanal nur geringen Einfluß auf den Gesamtdruckabfall hat: es handelt sich deshalb im wesentlichen um eine Strömung mit konstantem Druckabfall.

Die Möglichkeiten zur Berechnung solcher Strömungen sind noch sehr gering. Insbesondere weichen die theoretischen und experimentellen Ergebnisse für die Oszillationen in vielen bedeutenden Aspekten voneinander ab; während sich die vorherrschende Schwingungsfrequenz gut berechnen läßt, ist dies für die Schwingungsamplitude nicht möglich. Letztere ist jedoch für den Betrieb der Reaktoren von größter Bedeutung.

In der vorliegenden Arbeit wird gezeigt, daß man durch die Einführung einer zufallsverteilten Störung in den Wandwärmestrom Strömungsoszillationen erhält, welche erheblich besser mit den Versuchswerten übereinstimmen, als die nach früheren Methoden berechneten Werte.

КОЛЕБАНИЯ ПОТОКА В ПРОТОЧНЫХ СИСТЕМАХ КИПЕНИЯ С УСТАНОВИВШИМСЯ ПЕРЕПАДОМ ДАВЛЕНИЯ ПРИ СЛУЧАЙНОМ ВОЗБУЖДЕНИИ

Аннотация — Колебания потока в проточных системах кипения с установившимся перепадом давления характерны для кипящих водяных ядерных реакторов. Обычно геометрия реактора такова, что перемещающийся по нагретому каналу поток оказывает незначительное влияние на общий перепад давления, а следовательно, имеет место поток с установившимся перепадом давления. Расчеты потока в условиях установившегося перепада давления оставляют желать лучшего. В частности, осцилляции потока, полученные теоретически и экспериментально, значительно различаются по многим важным параметрам. В то время как доминирующая частота хорошо поддается расчету, амплитуду колебаний рассчитать невозможно, а именно она имеет большое значение при работе реактора. В настоящей работе показано, что случайное возбуждение теплового потока на стенке приводит к колебаниям потока; полученные результаты значительно лучше согласуются с экспериментальными данными, чем в ранее проведенных расчетах.